

Market Transaction Costs in Industrialization and Demographic Transition

Lewis S. Davis
Pierce Hall 107
Smith College
Northampton MA 01063

413-585-2792
lsdavis@email.smith.edu

Submitted to *Pacific Economic Review* June 28, 2005

Revised February 23, 2006

Abstract

This paper presents a unified theory of growth involving human capital accumulation, labor specialization, market expansion and falling fertility rates. The model suggests these processes, often analyzed separately, are intimately linked. The accumulation of specialized human capital increases the gains to labor specialization, leading agents to increase their participation in markets and reduce time spent at home. This raises the opportunity cost of child raising, lowering fertility rates. The model suggests a central role for market transaction costs in determining the timing and rate of fertility declines linked to rising income.

Key Words: Labor specialization, Market transaction costs, Industrialization, Fertility, Demographic transition.

JEL Classification: O1, O14, J13, D23, L14

I. Introduction

In many of the division of labor growth models that grew out of Yang and Borland's (1991) seminal article, producer-consumers face "make-or-buy" decisions in which gains to labor specialization are constrained by market transaction costs. The accumulation of

specialized capital goods increases the gains to labor specialization, driving a development process governed by increasing labor specialization, interpersonal trade and market expansion.

An important characteristic of these models that has not received much comment is that changes in labor specialization and market structure are accompanied by complementary changes in the organization of the household.¹ In particular, specialization and market expansion coincide with decreases in the share of resources devoted to producing goods for own-consumption. Furthermore, with the formation of firms (Yang and Borland, 1995, Davis, 2003) to help coordinate the division of labor, labor specialization results in production moving out of the domestic sphere. This paper investigates how these changes in household structure influence, and are influenced by, parental decisions regarding fertility and child education.

The model is built around a number of simple assumptions. Adults allocate time and human capital to the domestic and market sectors, informally “home” and “work,” to maximize a utility function that depends on their own consumption as well as the number and education of their children. Production in the market sector is subject to gains to labor specialization. The ability to trade with other specialists allows each market sector participant to concentrate her resources on a narrow range of intermediate goods, increasing the utilization rate of specialized human capital.

In contrast, in the domestic sector production is characterized by constant returns to scale. An adult who produces in the domestic sector does not coordinate with other producers and, therefore, forgoes possible gains to labor specialization. Market transaction costs play a central role in determining the trade-off between domestic and market sector production. In an economy

¹ See, for example, a recent review of literature on the division of labor by Cheng and Yang (2004).

with high market transaction costs, it is more difficult to coordinate production across market sector specialists, making domestic sector production more attractive.

Raising a child requires inputs of goods and parental time. These are substitutes, such that the more time a parent spends at home the lower is the cost of child rearing in terms of goods. Thus, while they forgo gains to labor specialization, agents that produce in the domestic sector enjoy lower costs of child raising, and this gain is larger for adults with larger families. A parent endows each child with an educational bequest that determines the level of human capital in the next generation. Since parents value both the number and educational level of their children, they face a familiar “quality-quantity” trade-off in making reproductive decisions.

The model support three dynamic equilibria. If market transaction costs are sufficiently high, the model supports an “autarkic” equilibrium in which interpersonal trade is absent. All production occurs at home, labor is unspecialized, families are large, and children are poorly educated.

A sufficient fall in market transaction costs allows an economy to escape this low level equilibrium, entering a transitional stage of development in which the evolution of the economy is driven by rising human capital bequests. The accumulation of specialized human capital increases the gains to labor specialization, causing adults to allocate more time to market production. In the market sector, this results in increases in labor specialization, market size, interpersonal trade and output. Simultaneously, the fall in time allocated to the domestic sector increases the cost of raising a child. Adults respond by having fewer children and educating each child more. Intuitively, as the fixed cost of a child rises, parents increase the utilization of each child. Viewed over successive generations, rising levels of human capital generate both industrialization and a demographic transition. Falling fertility rates imply this transitional stage

eventually comes to an end, and the economy converges on a steady state in which population size, labor specialization and income are constant.

As this discussion suggests, a primary contribution of the analysis is to link two aspects of development that are generally analyzed separately. Labor specialization drives both market expansion and the falling fertility rates that mark the demographic transition. From a policy perspective, the model demonstrates the importance of market transaction costs for fertility decisions. The level of market transaction costs determine whether or not an economy becomes trapped in the high fertility, autarkic equilibrium. In addition, in a transitional economy, a fall in market transaction costs increases the gains to labor specialization, increasing time allocated to market production and reducing fertility rates. By implication, in developing countries, building better roads or reducing corruption may help reduce fertility rates.

In the immense empirical literature on the economics of fertility, prominent hypotheses link fertility decisions to the opportunity cost of children, generally proxied by female education and labor market participation.² This corresponds nicely with the model's emphasis on roles for human capital and non-domestic production. The model, however, treats education and market production opportunities as endogenous variables that, along with fertility rates, are influenced by market transaction costs. Thus, the analysis presented here suggests that empirical studies of fertility rates should attempt to include variables that capture exogenous variation in market transaction costs, such as the quality of physical infrastructure and the efficiency of commercial law. The commonly reported negative correlation between female education levels and fertility rates may reflect, in part, both variables' dependence on market transaction costs.

This paper also contributes to the rapidly growing literature that attempts to present an integrated theory linking the demographic and economic transitions that occur during

industrialization. In the main line of this research, characterized by Hansen and Prescott (2002), Jones (2001), Tamura (2002) and Galor and Weil (2000), technological progress raises the return to skilled labor, simultaneously increasing output and inducing a demographic transition.³

The analysis presented here contributes to this literature by suggesting a central role for market transaction costs. Transaction costs reflect a number of underlying factors, such as geography and physical infrastructure, but they are also closely linked to institutional quality, particularly the protection of property rights and enforcement of private contracts (North, 1990, Knack and Keefer 1995). In contrast with technology, which is often understood to be a global public good, the institutional determinants of market transaction costs are held to vary dramatically across countries. While the growth of human knowledge may have played an important role in historical fertility declines, theories that suggest an important role of institutions in determining fertility rates are better suited to account for a contemporary world characterized by dramatic cross-country fertility differentials.

II. The Model

Consider an economy with N identical households. A household consist of a single adult and n children. Adults live for a single period or generation, indexed by $\tau = \{0, 1, \dots\}$.

Reproduction is asexual, and children make no economic decisions.

Preferences

² See for example the special issue of *World Development* introduced by Murthi (2002).

³ Other approaches emphasize changes in gender roles (Galor and Weil, 1996) and the return to child labor (Doepke, 2004).

The utility of an adult in generation τ is adapted from Becker and Barro (1988) and Becker et al. (1990): $U_\tau = \frac{c_\tau^\sigma}{\sigma} + a(n_\tau)n_\tau \frac{h_{\tau+1}^\sigma}{\sigma}$. Utility is increasing in adult consumption c_τ , the number of children n_τ , and altruistic bequests that take the form of investments in their children's human capital $h_{\tau+1}$. The degree of altruism per child, $a(n)$, is assumed to decrease in the number of children according to $a(n) = \theta n^{-\varepsilon}$. It follows that the weight placed on per child bequests, which may be interpreted as an endogenous discount rate, rises with the number of children but at a decreasing rate. Utility is given by

$$(1) \quad U_\tau = \frac{c_\tau^\sigma}{\sigma} + \theta n_\tau^{1-\varepsilon} \frac{h_{\tau+1}^\sigma}{\sigma},$$

where $2\varepsilon + \sigma < 1$. As noted below, this parameter restriction rules out explosive outcomes in which an adult leaves an infinite human capital bequest to a vanishingly small number of children.

The assumption that parents value bequests rather than, say, child utility or consumption, simplifies the analysis by making parental decisions independent of future economic outcomes. This facilitates the analysis of the transition dynamics that are our primary interest. To reduce notational clutter, the generational subscript τ is omitted from variables when it is not required to differentiate between generations.

Resource Endowments and Sectoral Allocations

A representative agent is endowed with one unit of time and h units of human capital. These resources may be used to produce goods in either of two sectors, a domestic sector and a market sector. That is, they may produce either “at home” or “at work.” Individual resource constraints imply

$$(2) \quad \begin{aligned} t_D + t_M &= 1, & t_M, t_D &\geq 0 \\ h_D + h_M &= h, & h_M, h_D &\geq 0 \end{aligned}$$

where subscripts D and M refer to the domestic and market sectors, respectively. In the remainder of the paper, we will use the terms “market” and “domestic” to refer to sectoral activities or values. Thus, for example, an agent’s domestic production is the value of output she produces in the domestic sector, and her market time is the time she allocates to production in the market sector.

Production in the Market Sector

Production in the market sector is subject to gains to labor specialization. There is a large number of intermediate goods that are produced using specialized skills. By concentrating their educational resources on a narrow range of intermediate goods, producers enjoy higher level of skill in each activity and are able to exploit their skills more intensively (Rosen 1983). The counterpart of specialization, however, is interdependence. Specialized agents must trade with other specialists to obtain those intermediates that they do not themselves produce, and such trading is assumed to be costly. Consequently, an agent’s ability to take advantage of gains to labor specialization is limited by market transaction costs.

In the market sector agents produce intermediate goods. There is a continuum of intermediate goods indexed by $a \in [0,1]$, all of which must be combined to produce the final good. Intermediate good output is given by

$$(3) \quad z_a = A t_a h_a^\alpha,$$

where $\alpha \in (0,1)$ and t_a and h_a are inputs specific to intermediate good a . Note that the exponents in (3) sum to more than one, indicating increasing returns in the production of intermediate

goods. It is this characteristic of production that gives rise to gains to labor specialization in the market sector.

Labor specialization is inversely related to the number of intermediates an agent produces, w . In particular, labor specialization is given by $s = I/w$. Market sector resources are allocated uniformly across intermediate goods. It follows that an agent's market output, z_M , is increasing in her labor specialization:

$$(4) \quad z_M = wz_a = wA \left(\frac{t_M}{w} \right) \left(\frac{h_M}{w} \right)^\alpha = As^\alpha t_M h_M^\alpha.$$

Market Transaction Costs and Optimal Labor Specialization

Intermediate goods are complements in the production of a composite good that may be used either for consumption or investment and is taken as the numeraire. One unit of the composite good is produced in Leontief fashion by combining one unit of each of the intermediate goods. The strong complementarity embodied in the final good technology requires that a specialized producer must trade with other specialists to obtain those intermediate goods she does not produce herself. We call an interpersonal trading group a market. Market size is given by the number of trade partners, which equals each member's specialization: $m = s$. If each agent produces one tenth of the intermediate goods, each market must have ten trade partners. This implies labor specialization cannot exceed population size: $s \leq N$.

Market transactions are assumed to be costly. Market transaction costs may reflect transportation costs (Smith, 1776/1976), principle-agent conflicts (Becker and Murphy, 1992), information costs (Coase, 1991, and Arrow, 1974), and contracting costs (Williamson, 1979), all of which reflect more fundamental institutional and technological characteristics of an economy.

Market transaction costs are assumed to rise in the number of an agent's trade partners,

$$x(m) = bm^\gamma, \text{ where } \gamma \geq 1 \text{ and } b > 0.$$

Symmetry implies that intermediates are traded one-for-one, such that the quantity of the final good an agent produces equals the value of her intermediate goods less transaction costs:

$$(5) \quad y_M = As^\alpha t_M h_M^\alpha - bs^\gamma.$$

Other than the cost of trading with other specialists, the assembly of the final good is assumed to be costless. By implication, we address horizontal division of labor across intermediate goods, but not the vertical division of labor between an intermediate good and final good sector. In making this assumption, we follow the main line of research on labor specialization (Becker and Murphy, 1992, Yang and Borland, 1991, Tamura 1992). The final good serves as a numeraire. Because it is costlessly assembled from a unit continuum of intermediate goods, the relative (shadow) price of intermediate goods is one.

Given their market sector allocations, agents choose labor specialization to maximize (5).

Ignoring for the moment the upper limit on labor specialization, $s \leq N$, we have

$$(6) \quad \begin{aligned} s^* &= \left[\frac{\alpha A}{\gamma b} \right]^{\frac{1}{\gamma-\alpha}} t_M^{\frac{1}{\gamma-\alpha}} h_M^{\frac{\alpha}{\gamma-\alpha}} \\ y_M &= \left[\frac{\gamma-\alpha}{\gamma} \right] \left[\frac{\alpha A}{\gamma b} \right]^{\frac{\alpha}{\gamma-\alpha}} A t_M^{\frac{\gamma}{\gamma-\alpha}} h_M^{\frac{\alpha\gamma}{\gamma-\alpha}} \end{aligned}$$

The equations in (6) capture the fundamental characteristics of production with gains to specialization. A fall in the level of market transaction costs, b , increases labor specialization and, consequently, market income. The exponents on labor and human capital have also increased. This reflects Rosen (1983) and Young's (1928) argument that specialization allows agents to exploit task-specific inputs more intensively.

In the remainder of the paper we adopt parameter restrictions that maintain the primary characteristics of equation (6) while simplifying the analysis. First, we assume $\alpha = \gamma(1 - \alpha)$. This implies that the gains to specialization, α , exactly offset rising transaction costs, γ , and diminishing returns to human capital, $1 - \alpha$. As a result, market production exhibits constant marginal returns to human capital. This is a requirement for endogenous growth and, thus, a common assumption in growth theory. We further assume $\alpha = 1/2$, and consequently $\gamma = 1$, to simplify some algebra later on without qualitatively changing the outcome of the model. Given these restrictions, labor specialization and market output are given by

$$(7a) \quad s^* = \min \left\{ \frac{A_M}{b} t_M^2 h_M, N \right\}$$

and

$$(7b) \quad y_M = \begin{cases} A_M t_M^2 h_M & s^* < N \\ AN^{1/2} t_M h_M^{1/2} & s^* = N \end{cases}$$

where $A_M = \frac{A^2}{4b}$. Equation (7b) provides a reduced-form expression for final good output in the market sector that is used in the optimization exercises below.

Production and Child Raising in the Domestic Sector

In the domestic sector producers are generalists. Agents do not trade with other producers and, therefore, do not take advantage of the gains to labor specialization. The final good production technology is linear in both time and human capital:

$$(8) \quad y_D = A t_D h_D,$$

where y_D is the quantity of the final good produced in the domestic sector. The absence of gains to specialization in domestic production is a simplification meant to capture the idea that opportunities for exploiting specialized skills are limited in domestic production.

While agents that produce “at home” do not enjoy gains to specialization, they have an advantage when it comes to raising children. In raising a child, parents incur a fixed cost ϕ . Both parental time and goods contribute to child raising according to $\phi = t_D f$, where t_D and f are parental time and goods devoted to child raising. It follows that the more time spent in domestic production, the lower the cost in goods of raising a child. Alternately, spending more time “at work” in the market sector increases the cost of raising a child:

$$(9) \quad f = \phi(1 - t_M)^{-1}.$$

Sectoral Resource Allocation and Organizational Outcomes

Due to non-convexities in the production function, it is useful to analyze sectoral resource allocation decisions separately, before proceeding to utility maximization below. We find that the allocation of time to market production increases in human capital and decreases in market transaction costs and family size. These outcomes reflect the trade-off between gains to specialization in the market sector and lower costs of child raising in the domestic sector. In addition, we derive a reduce-form equation relating final good to different stages of organizational development.

Parents allocate time and human capital to the domestic and market sectors to minimize the budget constraint, which is defined as

$$(10) \quad G(..) = c_\tau + n_\tau(f_\tau + h_{\tau+1}) - y_\tau$$

subject to the resource constraints in equation (2). To reduce notational clutter, hereafter we drop the sectoral subscripts on time allocations. We denote the time allocated to the market sector as “ t ” and time allocated to the domestic sector as “ $1-t$.”

With both market and domestic production linear in human capital, agents face an “either-or” decision regarding how they use their human capital: agents allocate their entire human capital stock to the sector with the highest return. Sectoral human capital returns depend, in turn, on how an agent allocates her production time between sectors. Let r_M and r_D be the return to human capital in the domestic and market sectors, respectively, and define \bar{t} as time in market production that equalizes these rates of return, $r_M = r_D$:

$$(11) \quad \frac{\bar{t}^2}{1-\bar{t}} = \frac{A}{A_M}.$$

The left hand side of (11) is increasing in \bar{t} and maps the unit interval onto the non-negative real numbers, $[0,1] \mapsto [0,\infty)$, implying the existence of a unique solution $\bar{t} \in (0,1)$ to (11) for any strictly positive sectoral productivity ratio.

Equation (11) defines a threshold level of time in market production. If time in the market exceeds this value, $t > \bar{t}$, then the return to human capital is higher in the market sector and $h_M = h$. Otherwise, $h_M = 0$. Differentiating (11), we find that the threshold level of time in the market is increasing in market transaction costs: $\bar{t} = \bar{t}^+(b)$. (11) is shown in *Figure 1* as the vertical line labeled $r_M = r_D$.

Next consider the allocation of productive time. If the threshold in (11) is not met, $t < \bar{t}$, then agents allocate all their human capital to domestic production. In this case, time in the market sector is unproductive, whereas time in the domestic sector both increases output and

reduces the cost of child raising. Thus, if $t < \bar{t}$ then agents allocate all their time to domestic production.

Alternately, if the threshold defined by (11) is met, an agent will not allocate all of her time to market production. This is because as an adult reduces time spent in the domestic sector, the resource cost of raising a child $f = \phi(1-t)^{-1}$ becomes infinite. More specifically, if $t > \bar{t}$, an agent allocates all of her human capital to market production, and her income is given by $y = A_M t^2 h$. Substituting this equation for output and (9) into the budget constraint, optimal time in market production is given by

$$(12) \quad t(1-t)^2 = \frac{\phi n_\tau}{2A_M h_\tau}.$$

The left hand side of (12) is decreasing in t for $t \in (1/3, 1)$, which is shown to hold below, indicating that the time allocated to market production is increasing in human capital and decreasing in family size and market transaction costs: $t = t(h/nb)^+$. Equation (12) is illustrated in *Figure 1* as a heavy solid line labeled $G_t = 0$. This locus indicates that agents allocate additional time to market production as human capital rises or family size falls.

As illustrated in *Figure 1*, (11) and (12) determine the allocation of time and human capital across sectors. The intersection of these two loci determines a threshold level of human capital $\bar{h} = \bar{h}(b, n)^+$. Below this level of human capital, an agent allocates her entire endowment of time and human capital to domestic production, as indicated by the heavy vertical line at $t = 0$. For levels of human capital above this threshold, the entire stock of human capital is devoted to market production and the allocation of time is determined by the $G_t = 0$ locus. At the onset of market production, time in the market is given by \bar{t} .

Provided human capital exceeds \bar{h} , (12) can be expressed in a more useful manner as

$$(13) \quad n_{\tau} f_{\tau} = \left[\frac{2(1-t)}{t} \right] y_M(t, h) = \psi [h/bn] y_M(t, h),$$

with $\psi = \frac{2(1-t)}{t} < 1$ being the share of resources allocated to raising children. Differentiating

(12), we see that a rise in human capital or fall in family size reduces the share of resources devoted to raising children, $\psi'(h/n) < 0$. ($\psi < 1$ implies $t > 2/3$, which implies the $G_{t_M} = 0$ locus in *Figure 1* has a positive slope.)

The dashed lines in *Figure 1* show the effects of a fall in the transaction cost coefficient, b . A fall in market transaction costs increases the return to human capital in the market, shifting the $r_M = r_D$ locus to the left. In addition, it raises the return to time in market production, shifting the $G_t = 0$ locus to the right. Both changes reduce the level of human capital necessary for market production: $\bar{h}' < \bar{h}$. For agents engaged in market production, time in the market also increases.

If the level of human capital is sufficiently high, then labor specialization will be constrained by population size. In this case, production is described by the second line of (7b) and (12) and (13) are replaced by

$$(12') \quad (1-t)^2 = \frac{\phi n}{AN^{1/2} h^{1/2}}$$

and

$$(13') \quad n_{\tau} f_{\tau} = \bar{\psi} [h/bn] AN^{1/2} t h^{1/2}$$

where $\bar{\psi} = \frac{(1-t)}{t}$ is the share of income devoted to raising children. Combining these with results from (7), (8), (12) and (13), we obtain an equation relating output to resource endowments:

$$(14) \quad y = \begin{cases} Ah & h < \bar{h} \\ A_M t^2 h & h > \bar{h}, s^* < N \\ AN^{1/2} t h^{1/2} & h > \bar{h}, s^* = N \end{cases}$$

As shown in the first line of (14), if human capital is sufficiently low, the gains to specialization are also low and agents opt for domestic production. In this case, production does not involve trade with other specialist producers, we refer to this as an *autarkic economy*. The lack of specialization and interpersonal trade in the autarkic economy suggests that this outcome corresponds to the traditional sector of a developing economy.

An economy that escapes from autarky, will enter a transitional phase of development. We refer to this as a *transitional economy* because, as detailed below, it experiences a number of mutually reinforcing processes that tend to result in this being a temporary organizational outcome. In a transitional economy, labor specialization is unconstrained, human capital is allocated to market production and output is given by the second line of (14). Economic organization is related to the level of human capital. An increase in the level of human capital results in greater labor specialization, larger markets and increased interpersonal trade. In addition, it increases the share of time allocated to market production and decreases the time allocated to parenting.

The third line of (14) refers to output in a mature economy. We refer to this as a mature economy because the gains available from market expansion and rapid increases in labor specialization that may generate increasing returns in a transitional economy are no longer

possible. In a mature economy labor specialization is constrained by population size rather than market transaction costs. Consequently, market size and labor specialization may only increase at the rate of population growth. Furthermore, with labor specialization independent of the level of human capital, production exhibits diminishing returns to capital.

Optimization and First Order Conditions

Parents choose consumption, family size, bequests and sectoral resource allocations to maximize (1) subject to resource constraints (2), the budget constraint (10), and the production technology (14). Substituting for the Lagrangian multiplier, the first order conditions for this problem are

$$(15) \quad L_{h_{\tau+1}} = 0: \quad \theta n_{\tau}^{1-\varepsilon} h_{\tau+1}^{\sigma-1} = c_{\tau}^{\sigma-1} n_{\tau}$$

$$(16) \quad L_n = 0: \quad (1-\varepsilon)\theta n^{-\varepsilon} \frac{h_{\tau+1}^{\sigma}}{\sigma} = c_{\tau}^{\sigma-1} (f_{\tau} + h_{\tau+1})$$

Equation (15) captures the trade-off between bequests and current consumption. Because this trade-off links individual outcomes across generations, it is at the heart of the model's dynamics. Rearranging (15), we have

$$(17) \quad h_{\tau+1} = \left[\theta n_{\tau}^{-\varepsilon} \right]^{\frac{1}{1-\sigma}} c_{\tau}$$

In larger families, the bequest received by each child is smaller relative to parental consumption. This result reflects the trade-off between the costs of bequests, which rise linearly with family size, and the weight of the bequest in the utility function, which rises less than linearly. The bracketed term in (17) may be interpreted as an endogenous discount rate, with the parents of larger families discounting each child's bequest more heavily. Alternately, the term $\left[\theta n_{\tau}^{-\varepsilon} \right]^{\frac{1}{1-\sigma}}$ may be interpreted as the price of adult consumption relative to human capital bequests. A rise

in family size increases the cost of providing a given level of education, making adult consumption more attractive.

Equation (16) describes the trade-off between current consumption and family size.

Dividing (16) by (15) gives the expression that drives the familiar quality-quantity trade-off:

$$(18) \quad h_{\tau+1} = \left[\frac{\sigma}{1 - \varepsilon - \sigma} \right] f_{\tau}.$$

Optimal fertility and bequest decisions imply that as raising a child becomes more expensive, parents choose to educate each child more. More formally, the term $\left[\frac{\sigma}{1 - \varepsilon - \sigma} \right]$ is the ratio of the utility derived from increasing the education of existing children relative to that of raising and educating an additional child. This trade-off plays a key role in the demographic transition. In particular, as adults spend more time at work, the cost of child raising rises and parents substitute education for family size. Intuitively, as the fixed cost of a child rises, parents utilize of each child more.

III. Analysis and Discussion

This section analyzes production and fertility decisions in each of the three possible organizational equilibria, autarky, transitional economies and mature economies. Particular attention is given to the conditions governing the existence and stability of an autarkic poverty trap, and the dynamic co-evolution of market and household organization in economies undergoing an economic and demographic transition. In addition, we highlight the interdependence of production and fertility decisions and their joint dependence on market transaction costs.

Output and Family Size in Autarky

If the initial level of human capital is sufficiently low, $h_0 < \bar{h}$, agents allocate all their resources to domestic production. Because domestic production does not involve interpersonal trade, we refer to this outcome as autarky. In autarky $t_D = 1$ and $h_D = h$. From equations (9) and (18), child raising costs and bequests are given by

$$(19) \quad \begin{aligned} f^A &= \phi \\ h^A &= \frac{\phi\sigma}{1-\varepsilon-\sigma}. \end{aligned}$$

The model supports a stable autarkic equilibrium provided $h^A < \bar{h}$, which implies that the children of autarkic parents will produce in autarky. The equilibrium bequest defines a unique value for family income

$$(20) \quad y^A = A \left[\frac{\phi\sigma}{1-\varepsilon-\sigma} \right].$$

Substituting these outcomes and (14) into (10) and (17) gives two equations relating consumption and family size:

$$(21) \quad c = \left[\frac{\phi}{1-\varepsilon-\sigma} \right] [\sigma A - (1-\varepsilon)n]$$

$$(22) \quad c = \left[\frac{\sigma\phi}{1-\varepsilon-\sigma} \right] \theta^{\frac{-1}{1-\sigma}} n^{\frac{\varepsilon}{1-\sigma}}$$

(21) reflects budgetary realities: raising and educating a larger family leaves fewer resources for consumption. (22) reflects the idea that increases in family size increase the marginal utility of bequests less than the cost, thus diverting more resources to consumption. As illustrated in *Figure 2*, together these relationships define equilibrium family size and adult consumption in the autarkic equilibrium.

Note that in autarky, the bequest is independent of parental human capital and income. Provided the autarkic equilibrium is stable, the transition to equilibrium values happens in a single generation. Furthermore, as shown in *Figure 2*, a positive productivity shock raises the income level, shifting the (21) locus upward resulting in higher consumption and larger families. That is, in the autarkic equilibrium family size and income vary directly.

Economic and Demographic Dynamics in a Transitional Economy

In a transitional economy, dynamic behavior is governed by two equations describing the relationship between bequests and adult consumption. The bequest-consumption ratio captures the fundamental trade-off between current and future generations, similar to the role of future-to-present consumption ratio in a Ramsey growth model. One relationship is found in equation

(17), which may be rearranged as $\frac{h_{\tau+1}}{c_{\tau}} = \theta^{\frac{1}{1-\sigma}} n_{\tau}^{\frac{-\varepsilon}{1-\sigma}}$. Equation (17) is shown as a concave line in $[h/c] - [1/n]$ space in *Figure 3*. Note that in *Figure 3*, family size decreases along the horizontal axis.

Substituting (13) and (18) into the budget constraint to eliminate y and f , gives a second such relationship:

$$(23) \quad \frac{h_{\tau+1}}{c_{\tau}} = \left[\frac{\sigma\psi}{(1-\varepsilon-\sigma)\sigma - (1-\varepsilon)\psi} \right] \left[\frac{1}{n_{\tau}} \right].$$

Equation (23) reflects the fact that having a larger family tends to increase the cost of bequests relative to current consumption. Furthermore, an increase in the share of income devoted to raising children, ψ , causes bequests to fall relative to consumption. As shown in the appendix the parameter restriction $2\varepsilon + \sigma < 1$ is sufficient to insure that (17) and (23) intersect and define unique equilibrium values for family size and the bequest-consumption ratio.

Together with equations (10), (13) and (18), these outcomes determine the remaining variables of interest:

$$(24) \quad c_\tau = \left[1 - \frac{(1-\varepsilon)\psi_\tau}{1-\varepsilon-\sigma} \right] y_\tau$$

$$(25) \quad n_\tau = \theta^{\frac{-1}{1-\varepsilon-\sigma}} \left[\frac{\sigma\psi}{(1-\varepsilon-\sigma)\sigma - (1-\varepsilon)\psi} \right]^{\frac{1-\sigma}{1-\varepsilon-\sigma}}$$

$$(26) \quad h_{\tau+1} = \frac{\left[(1-\varepsilon-\sigma)\sigma - (1-\varepsilon)\psi \right]^{\frac{1-\sigma}{1-\varepsilon-\sigma}}}{(1-\varepsilon-\sigma)(\sigma\psi)^{\frac{\varepsilon}{1-\varepsilon-\sigma}}} y_\tau.$$

These results allow us to trace the impact of an increase in human capital through the economy. An increase in human capital allows greater exploitation of gains to specialization in market production (7a), increasing the value of time in market production. With time in market production more productive, parents reduce the resources allocated to raising children ψ , as seen in (13).

This affects bequests and consumption through both substitution and income effects. The substitution effect is illustrated in *Figure 3*. The fall in ψ corresponds to a shift of (23) to the right, with the equilibrium moving from *E* to *F*, causing parents to opt for smaller families. Having fewer children reduces the cost of bequests relative to consumption, raising the bequest-consumption ratio. The income effect occurs because having smaller families frees up additional resources, allowing both consumption and bequests to rise relative to income, as seen in (24) and (26). Therefore, both income and substitution effects tend to increase the size of bequests.

To summarize, in a transitional economy, human capital accumulation increases time at work, income, consumption and bequests. Family size and the share of income devoted to

raising and educating children fall. Furthermore, parenting becomes more good intensive and less time intensive, as busy parents substitute goods for time in child raising.

By increasing the return to time spent in market production, a fall in market transaction costs, b , gives rise to a similar chain of interactions. As indicated by the heavy dashed lines in *Figure 1*, the fall in market transaction costs increases the allocation time to the market sector. The increase in market time results in a decrease in the share of income devoted to raising children, ψ , which corresponds to a rightward shift in (23) in *Figure 3*. As indicated by (22), this lowers family size and increases adult consumption and the bequest relative to income.

Growth in a Transitional Economy

Dividing both sides of (26) by h_t , we find that the growth of human capital is given by

$$(27) \quad 1 + g_h = \left[\frac{[(1 - \varepsilon - \sigma)\sigma - (1 - \varepsilon)\psi]^{\frac{1-\sigma}{1-\varepsilon-\sigma}}}{(1 - \varepsilon - \sigma)(\sigma\psi)^{\frac{\varepsilon}{1-\varepsilon-\sigma}}} \right] A_M t_M^2 .$$

The bracketed term in (27) is the share of the bequest in income and the remainder of the equation is the return to human capital in market production. Both terms are increasing in the level of human capital. First, a rise in human capital increases the time devoted to market production, as indicated in *Figure 1*. Second, the rise in market time induces a fall in family size, which increases bequests though income and substitution effects as discussed above.

A fall in market transaction costs affects the transitional growth rate in a similar fashion. By lowering the cost of market transactions, it increases labor specialization and, with it, the return to time in market production. Thus, the transitional growth rate may be expressed as

$g = g(h, b)$. Setting growth equal to zero in (26) implicitly defines a threshold level of human capital, $\hat{h} = \hat{h}(b)$, above which growth is positive and below which growth is negative.

We highlight the role of transaction costs by investigating human capital growth near the threshold from an autarkic to a transitional economy. Define $g_h(\bar{h}^+)$ and $g_h(\bar{h}^-)$ to be the growth rate of human capital as human capital approaches \bar{h} from above and below, respectively. In the appendix, it is shown that the growth rate of human capital experiences a positive jump at \bar{h} . Furthermore, $g_h(\bar{h}^+) > 0$ and $g_h(\bar{h}^-) > 0$ provided $h^A > \bar{h}(1 - \bar{t})$ and $h^A > \bar{h}$, respectively. As illustrated in *Figure 4*, these conditions define two threshold levels of the transaction costs coefficient, $b_1 > b_0 > 0$, such that $g_h(\bar{h}(b_1)^+) = 0$ and $g_h(\bar{h}(b_0)^-) = 0$. (See appendix for details.)

The level of the transaction cost coefficient relative to these two thresholds determines important aspects of dynamic performance. If transaction costs are sufficiently high, $b > b_1$, the threshold for positive growth is greater than that for a transition economy: $\hat{h} > \bar{h}$. Not only is the autarkic equilibrium stable, in that the children of autarkic parents will produce in autarky, constituting a low-level trap, but the trap extends to include some transition economies. A transition economy with a low level of human capital will experience negative growth and falling human capital levels until $h_t < \bar{h}$ and the economy enters autarky. If the initial level of human capital is sufficiently high, $h_0 > \hat{h}$, the economy experiences positive growth, rising incomes and falling family size. The first case is illustrated in *Figure 5A*.

With transaction costs at an intermediate level, $b_1 > b > b_0$, the autarkic equilibrium is stable and constitutes a poverty trap. However, once started, the interlinked processes of labor specialization, human capital accumulation, falling family size and market expansion that

characterize a growing transitional economy are self-sustaining. An economy experiences positive and accelerating growth provided its initial human capital is sufficient to allow market production. The second case is illustrated in *Figure 5B*.

The third case, illustrated in *Figure 5C*, involves an economy with low transaction costs, $b < b_0$. In this case the autarkic equilibrium is not stable. An economy with a low initial endowment of human capital will produce in autarky for one generation. The subsequent generation will have sufficient human capital to enjoy positive growth in a transition economy.

These three cases illustrate one of the central contentions of the paper: for given initial conditions, the level of market transaction costs simultaneously determines whether an economy enters a dynamic equilibrium with rising income and education levels, expanding markets, growing labor specialization and falling fertility rates or is drawn to a low level, static equilibrium with low incomes and large families. Furthermore, if transaction costs are sufficiently low, the economy is drawn into the dynamic equilibrium regardless of its initial human capital endowment.

Mature Economies and Steady State Growth

The primary focus of this paper has been the role of market transaction costs in onset of industrialization and the demographic transition. In this subsection, we briefly account for the forces that eventually curtail the process of accelerating growth and falling family size described above. In a growing transition economy, human capital and income grow at an accelerating rate. Equation (7a) implies that market size and labor specialization also expand at an accelerating rate. On the other hand, the rate of population growth is falling, $g_N = n - 1$. Together, these outcomes imply that optimal market size will eventually outstrip population size. We refer to

such an economy as an (organizationally) mature economy in that process of accelerating growth through labor specialization and rapidly expanding markets has been exhausted.

With the gains to labor specialization limited by population size, $s^* = N$, production in a mature economy exhibits diminishing marginal returns to human capital, $y_M = AN^{1/2}th^{1/2}$. It follows that accumulation alone cannot drive growth. Income growth requires persistent population growth, which permits the exploitation of additional gains to specialization. The evolution of population size and human capital are given by

$$(28) \quad N_{\tau+1} = \theta^{\frac{-1}{1-\varepsilon-\sigma}} \left[\frac{\sigma \bar{\psi}}{(1-\varepsilon-\sigma)\sigma - (1-\varepsilon)\bar{\psi}} \right]^{\frac{1-\sigma}{1-\varepsilon-\sigma}} N_{\tau}$$

and

$$(29) \quad h_{\tau+1} = \left[\frac{[(1-\varepsilon-\sigma)\sigma - (1-\varepsilon)\bar{\psi}]^{\frac{1-\sigma}{1-\varepsilon-\sigma}}}{(1-\varepsilon-\sigma)(\sigma \bar{\psi})^{\frac{\varepsilon}{1-\varepsilon-\sigma}}} \right] A t \left[\frac{N_{\tau}}{h_{\tau}} \right]^{1/2} h_{\tau}$$

where $\bar{\psi} = \left[\frac{1-t}{t} \right]$ as in (13'). These equations are the mature economy counterparts to (25) and

(26).

The steady state is defined by values of human capital and population size such that $N_{\tau} = N_{\tau+1}$ and $h_{\tau} = h_{\tau+1}$ in (28) and (29) such that $g_h = g_N = 0$. Positive steady state growth may be ruled out as follows. If human capital is increasing in the steady state, then time in the market is also rising and, from (28), family size is falling, a violation of the steady state requirement that population growth be constant.

Setting $N_{\tau} = N_{\tau+1}$ in (28) gives the $g_N = 0$ locus, which is a vertical line at $h = h^{SS}$ in h - N space as shown in *Figure 6*. For $h > h^{SS}$, time in the market is higher than the steady state level, each adult has less than one child and population size falls. For $h < h^{SS}$, time in the

market is less than the steady state level, each adult has more than one child and population rises. The $g_h = 0$ locus is found by imposing $h_\tau = h_{\tau+1}$ in (29). This locus is shown as an upward-sloping curve in *Figure 6*. Equation (29) indicates that the growth rate of human capital is increasing in population size and decreasing in human capital. It follows that human capital is increasing above the $g_h = 0$ locus and decreasing below it. The steady state is stable, as shown in the appendix.

IV. Conclusion

This paper has outlined a transaction cost-based theory of industrialization and demographic transition. The level of market transaction costs determines the range of initial conditions for which an economy is caught in a poverty trap with high fertility rates and low levels of income and human capital. An economy that avoids this trap enters a transitional phase of development characterized by human capital accumulation, labor specialization, market expansion, increased interpersonal trade and rising incomes. Simultaneously, time allocated to the domestic sector falls, coinciding with a fall in fertility and the share of income devoted to raising and educating children, and increases in educational bequests and the good-intensity of child-raising. The model also supports a stable, steady state equilibrium for mature economies.

In addition to influencing whether an economy is in an autarkic or transitional phase of development, transaction costs directly affect fertility decisions during the demographic transition. In particular, a fall in market transaction costs increases the return to time in market production, which raises the opportunity cost of time spent raising children and lowers desired fertility. The central role of market transaction costs in the model may be interpreted as

suggesting the importance of physical infrastructure and institutional quality for fertility decisions. By encouraging agents to engage production for the market, good roads and good laws may increase household incomes and simultaneously reduce the incentive to have large families.

Appendix

A solution to equation (25) exists provided (23) intersects (17). This is non-obvious because ψ , appearing on RHS of (23) is itself a function of family size. A sufficient condition for this intersection to occur is that (23) exceeds (17) as family size falls. We consider this by examining the ratio of the right hand side of (23) to that of (17). Ignoring multiplicative constants, as n goes to zero this ratio approaches $\frac{\psi[h/n]}{n^{1-\sigma}}$. Substituting from (19), this equals the

limit of $\frac{2(1-t)}{t} [t(1-t)^2]^{\frac{1-\varepsilon-\sigma}{1-\sigma}}$ as $t \rightarrow 1$. This limit is unbounded provided $2 \left[\frac{1-\varepsilon-\sigma}{1-\sigma} \right] > 1$ or

$2\varepsilon + \sigma < 1$, as reported in the text.

Recalling that \bar{h} and \bar{t} are functions of the transaction cost coefficient, we define b_0 and b_1 such that $g_h(\bar{h}^-) = 0$ and $g_h(\bar{h}^+) = 0$, respectively. It remains to be shown that b_0 and b_1 exist and that $b_0 < b_1$. Equation (11) defines the threshold level of time in market production to

be increasing in the transaction cost coefficient: $\bar{t} = \bar{t}^+(b)$. Substituting this and $A_M = \frac{A^2}{4b}$ into

(12), the threshold level of human capital is given by $\bar{h}(b) = \frac{4b\phi n}{2A^2 f(\bar{t}(b))}$, where

$f(\bar{t}(b)) = \bar{t}(b)[1 - \bar{t}(b)]^2$. For acceptable levels of time in the market, $\bar{t} \in (2/3, 1)$, we have

$f'(\bar{t}) < 0$, which implies $f_b = f'(\bar{t})\bar{t}'(b) < 0$ and $\bar{h}_b > 0$. Noting also that $\bar{h}(b)$ is a strictly

monotonic, continuous real function that maps onto the positive real numbers, and that $h^A \in \mathfrak{R}^+$,

it follows that there exists a unique value of b , $b = b_0$, such that $\bar{h}(b_0) = h^A$, which implies

$g_h(\bar{h}(b_0)^-) = 0$. For $b < b_0$, $\bar{h}(b_0) < h^A$ implying that the autarkic equilibrium is unstable.

Similarly, for $b > b_0$, $\bar{h}(b_0) > h^A$ implies that the autarkic equilibrium is stable.

The proof of the existence of b_1 such that $g_h(\bar{h}(b_1)^+) = 0$ is similar and relies on showing

that $q(b) \equiv \bar{h}(b)[1 - \bar{i}(b)]$ is increasing in b . From (12) we have $q(b) = \frac{4bn\phi}{2A^2 p(b)}$, where

$p(b) = \bar{i}(b)[1 - \bar{i}(b)]$. Noting that $p'(b) < 0$ for $\bar{i} \in (2/3, 1)$, we have $q'(b) > 0$. The nice

properties of $q(b)$ indicate the existence of a unique value of b , $b = b_1$, such that $q(b_1) = h^A$,

which implies $g_h(\bar{h}(b_1)^+) = 0$. It follows that $g_h(\bar{h}(b)^+) < 0$ for $b > b_1$, and $g_h(\bar{h}(b)^+) > 0$ for

$b < b_1$. The relative magnitudes of the two thresholds, $b_0 < b_1$, follows directly from

$$\bar{h}(b) > \bar{h}(b)[1 - \bar{i}(b)].$$

Next we address the stability of the mature economy steady state. Define

$$F(N_\tau, h_\tau) = \theta^{\frac{-1}{1-\varepsilon-\sigma}} \left[\frac{\sigma \bar{\psi}}{(1-\varepsilon-\sigma)\sigma - (1-\varepsilon)\bar{\psi}} \right]^{\frac{1-\sigma}{1-\varepsilon-\sigma}}$$

and

$$G(N_\tau, h_\tau) = \left[\frac{[(1-\varepsilon-\sigma)\sigma - (1-\varepsilon)\bar{\psi}]^{\frac{1-\sigma}{1-\varepsilon-\sigma}}}{(1-\varepsilon-\sigma)(\sigma \bar{\psi})^{\frac{\varepsilon}{1-\varepsilon-\sigma}}} \right] At \left[\frac{N_\tau}{h_\tau} \right]^{1/2},$$

and employ a change of variable $\tilde{N}_\tau = N_\tau - N^{SS}$ and $\tilde{h}_\tau = h_\tau - h^{SS}$. Using a Taylor expansion,

near the steady state (28) and (29) may be approximated by

$$\begin{bmatrix} \tilde{N}_{\tau+1} \\ \tilde{h}_{\tau+1} \end{bmatrix} = \begin{bmatrix} F_N(N^{SS}, h^{SS}) & F_h(N^{SS}, h^{SS}) \\ G_N(N^{SS}, h^{SS}) & G_h(N^{SS}, h^{SS}) \end{bmatrix} \begin{bmatrix} \tilde{N}_\tau \\ \tilde{h}_\tau \end{bmatrix} = A \begin{bmatrix} \tilde{N}_\tau \\ \tilde{h}_\tau \end{bmatrix}.$$

This first-order system of difference equations is stable provided $tr(A) = F_N + G_h < 0$ and $\det(A) = F_N G_h - G_N F_h > 0$. Noting that $F_N = 0$ and $G_N > 0$, we must show $G_h < 0$ and $F_h < 0$.

$F_h < 0$ follows unambiguously from $\bar{\psi}_h < 0$. Using (12'): $(1-t)^2 = \frac{\phi n}{AN^{1/2}h^{1/2}}$, we rewrite

$$G(..) \text{ as } G(N_\tau, h_\tau) = \left[\frac{\sigma}{1-\varepsilon-\sigma} \right] \left[\frac{1-t}{tn} \right] At \left[\frac{N_\tau}{h_\tau} \right]^{1/2} = \left[\frac{\sigma\phi^{1/2}}{1-\varepsilon-\sigma} \right] \left[\frac{A^{1/2}N^{1/4}}{n^{1/2}h^{3/4}} \right]. \text{ Next, differentiate}$$

(28) with respect to n and h , and impose steady state condition $n = 1$ to obtain:

$$-\frac{dn}{dh} = \frac{\bar{\psi}_h}{\bar{\psi}_n - \frac{\bar{\psi}_h^2}{(1-\sigma)\theta^{1-\sigma}}} < \frac{\bar{\psi}_h}{\bar{\psi}_n} = \frac{n}{2h}. \text{ This implies } -\left[\frac{dn}{dh} \right] \frac{h}{n} < \frac{3}{2}, \text{ which implies } G_h < 0.$$

References

- Arrow, Kenneth J. (1974). *The Limits to Organization*. New York: Norton.
- Becker, Gary S. and Robert J. Barro (1988) "A Reformulation of the Economic Theory of Fertility," *Quarterly Journal of Economics* 103(1), 1-25.
- Becker, Gary S., Kevin M. Murphy and Robert Tamura (1990), "Human Capital, Fertility and Growth," *Journal of Political Economy* 98(5), S12-37.
- Becker, Gary S., and Kevin M. Murphy. (1992). "The Division of Labor, Coordination Costs, and Knowledge," *Quarterly Journal of Economics*, 57(4), pp. 1137-1160.
- Cheng, Wenli and Xiaokai Yang (2004) "Inframarginal Analysis of the Division of Labor: A Survey," *Journal of Economic Behavior and Organization* 55(2), 137-174.
- Coase, Ronald H. (1991). "The Nature of the Firm," in *The Nature of the Firm*, Edited by Oliver E. Williamson and Sidney G. Winter. New York: Oxford University Press.
- Davis, Lewis S. (2002) "Institutional Structure, the Division of Labor and Technological Catching Up," manuscript, Smith College.
- Davis, Lewis S. (2003) "Toward a Unified Transaction Cost Theory of Economic Organization," forthcoming, *Journal of Institutional and Theoretical Economics* 159(3), 571-93.

- Doepke, Matthias (2004) "Accounting for Fertility Decline During the Transition to Growth," *Journal of Economic Growth* 9, 347-383.
- Galor, Oded and David N. Weil (1996) "The Gender Gap, Fertility and Growth," *American Economic Review* 86(3), 375-387.
- Galor, Oded and David N. Weil (2000) "Population, Technology and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond," *American Economic Review* 90(4), 806-828.
- Hansen G. D. and Edward C. Prescott (2002) "Malthus to Solow," *American Economic Review* 92(4), 1205-1217.
- Jones, Charles I. (2001) "Was the Industrial Revolution Inevitable? Economic Growth over the Very Long Run," *Advances in Macroeconomics* 1(2), Article 1.
- Knack, Stephen and Keefer, Philip (1995) "Institutions and Economic Performance: Cross-Country Tests Using Alternative Institutional Measures," *Economics and Politics* 7 (3), 207-227.
- Murthi, Mamta (2002) "Fertility Change in Asia and Africa," *World Development* 30 (10), 1769-78.
- North, Douglass C. (1990). *Institutions, Institutional Change and Economic Performance*, New York: Cambridge University Press.
- Rosen, Sherwin. (1983). "Specialization and Human Capital," *Journal of Labor Economics* 1(1), 43-49.
- Smith, Adam. (1776/1976). *An Inquiry into the Nature and Causes of the Wealth of Nations*. Edited by Edwin Cannan. Chicago: University of Chicago Press.
- Tamura, Robert. (1992). "Efficient Equilibrium Convergence: Heterogeneity and Growth," *Journal of Economic Theory*, 58, pp. 355-376.
- Tamura, Robert (2002) "Human Capital and the Switch from Agriculture to Industry," *Journal of Economic Dynamics and Control* 27(2), pp. 207-42.
- Williamson, Oliver E. (1979). "Transaction-Cost Economics: The Governance of Contractual Relations," *Journal of Law and Economics* 22, pp. 233-261.
- Yang, Xiaokai and Jeff Borland (1991) "A Microeconomic Mechanism for Economic Growth," *Journal of Political Economy* 99(3), pp. 460-482.

Yang, Xiaokai and Jeff Borland (1995), "Specialization, Product Development, Evolution of the Institution of the Firm, and Economic Growth," *Journal of Evolutionary Economics*, 5, 19-42.

Young, Allyn A. (1928). "Increasing Returns and Economic Progress," *Economic Journal* 38, 527-42.

Figure 1: Resource allocation in a transitional economy

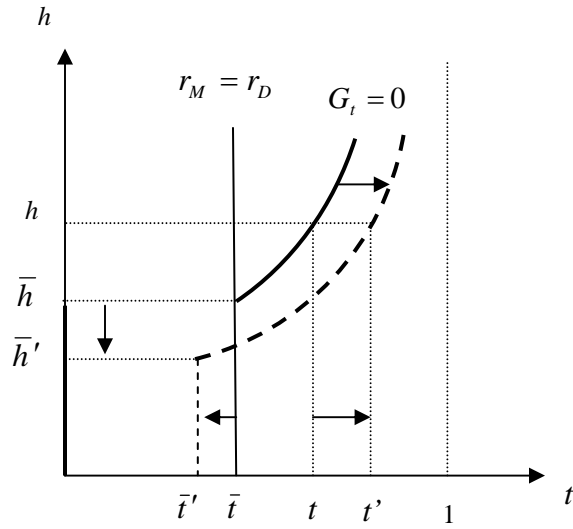


Figure 2: Consumption and Family Size in Autarky

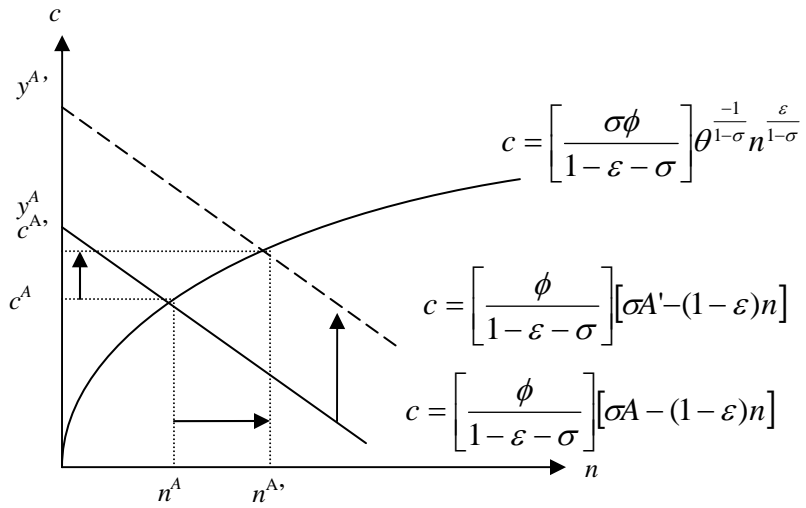


Figure 3: Equilibrium in a Transitional Economy

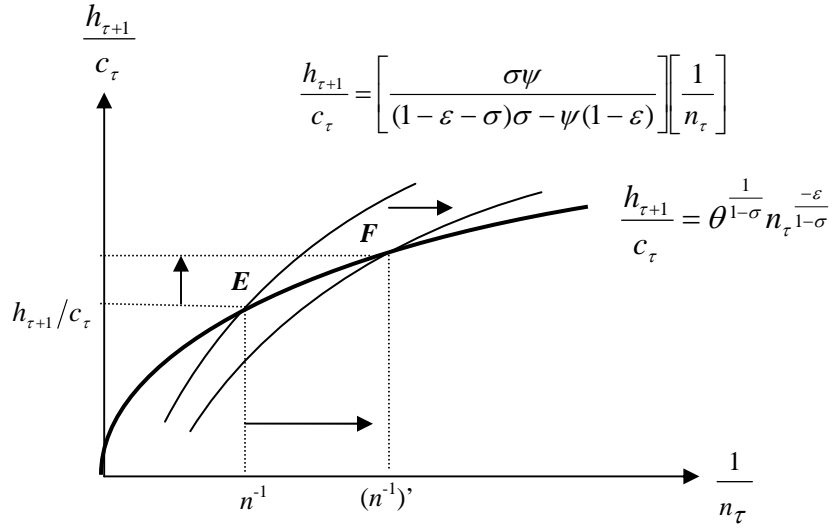


Figure 4: Transaction Cost Thresholds

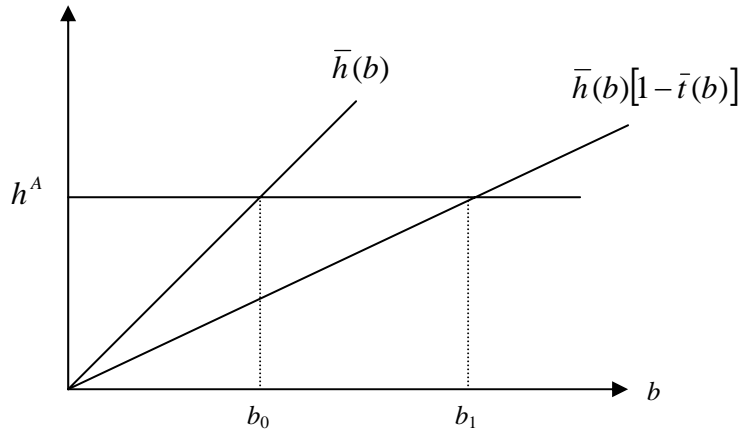


Figure 5a: The Evolution of Human Capital, $b > b_1$

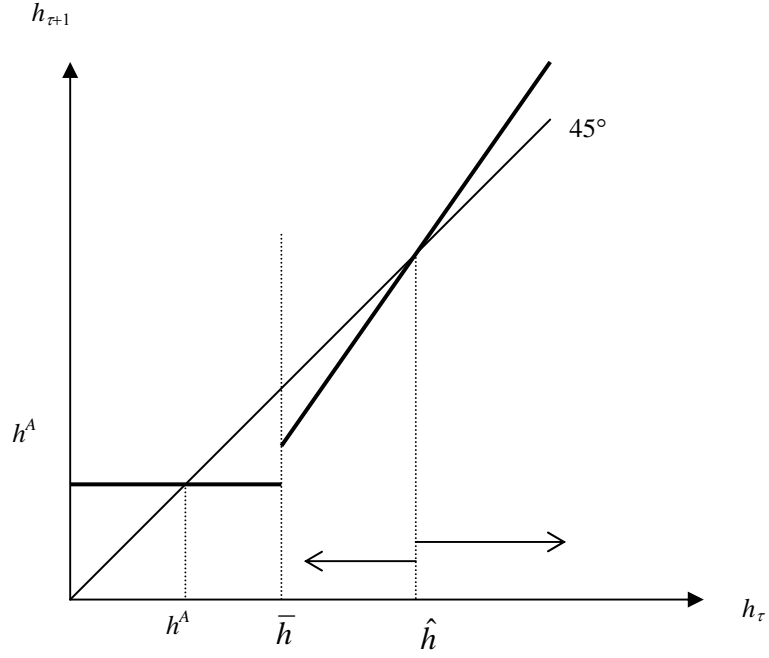


Figure 5b: The Evolution of Human Capital, $b_1 > b > b_0$

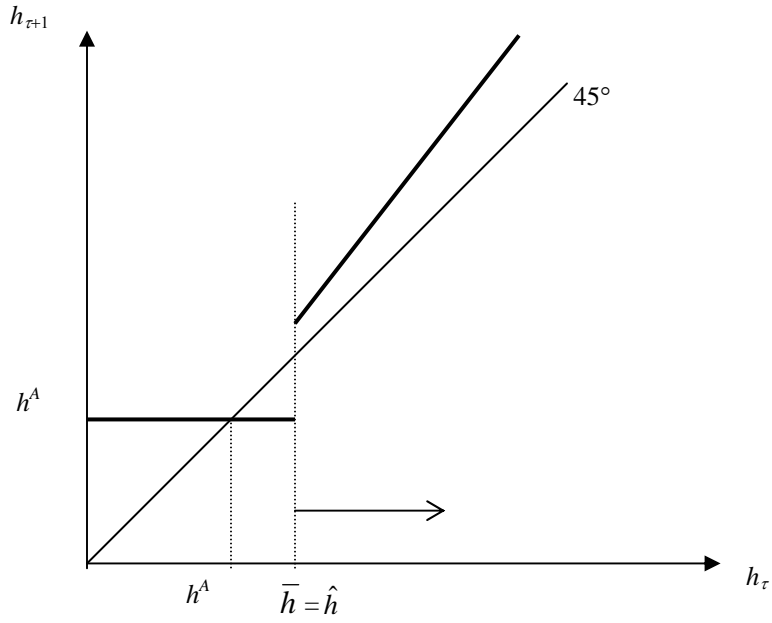


Figure 5c: The Evolution of Human Capital, $b < b_0$

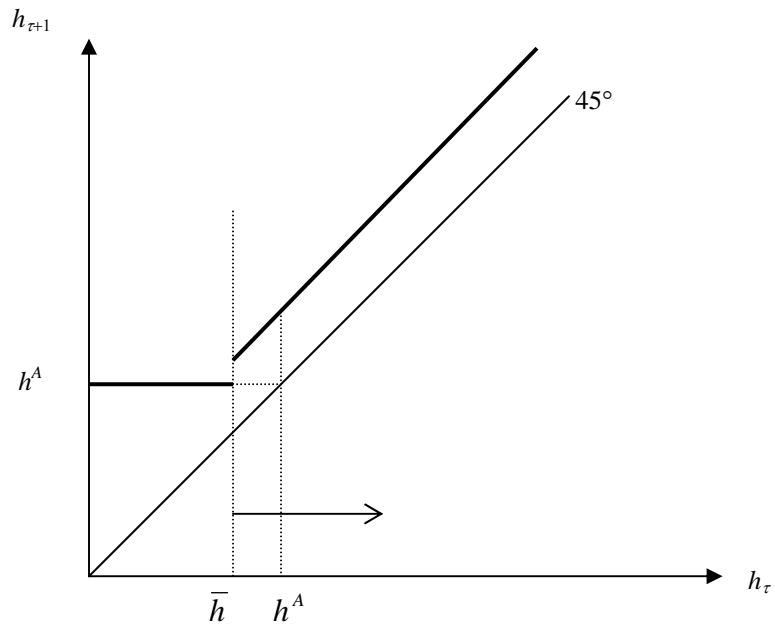


Figure 6: Steady State in a Mature Economy

